M.Sc. (Mathematics) (New CBCS Pattern) Semester-II

PSCMTHT10A - Optional Paper : Differential Geometry

P. Pages : 2 | GUG/S/25/13750

Time : Three Hours | Max. Marks : 100

Notes : 1. Solve all questions.
2. Each questions carry equal marks.

UNIT – I

1. a) Show that a proper parametric transformation either leaves every normal unchanged or reverses the direction of the normal. **10**

b) Show that a metric is invariant under a parametric transformation. **10**

OR

c) Show that the parameters on a surface can always be chosen so that the curves of the given family & the orthogonal trajectories become parametric curves. **10**

d) On the paraboloid $x^2 = y^2 = z$ find the orthogonal trajectories of the section by the planes z = constant. **10**

UNIT – II

2. a) If U & V are in $U\frac{\partial T}{\partial v} - V\frac{\partial T}{\partial \dot{u}} = 0$ then show that $\dot{u}U + \dot{v}V = \frac{dT}{dt}$ **10**

b) Show that the curves of the family $\frac{v^3}{u^2} =$ constant are geodesics on a surface with the metric $v^2du^2 = 2uvdudv + 2u^2dv^2, u > 0,\ v > 0$ **10**

OR

c) Show that for any curve on a surface the geodesic curvature vector is intrinsic. **10**

d) State & prove the Gauss-Bonnet theorem. **10**

UNIT – III

3. a) Find the 2nd fundamental form for the general surface of revolution. $r = (g(u)\cos v,\ g(u)\sin v, f(u))$ **10**

b) Show that the points of the paraboloid $r = \left(u\cos v,\ u\sin v, u^2\right)$ are elliptic but the points of helicoid $r = (u\cos v, u\sin v, av)$. are hyperbolic. **10**

OR

c) Show that all points on a sphere are umbilics. **10**

d) Show that the edge of regression of the polar developable of a space curve is the locus of the centers of spherical curvature. **10**

UNIT – IV

4. a) If the lines of curvature are parametric curves then show that the Codazzi equations are **10**

$$L_2 = \frac{1}{2}E_2\left(\frac{L}{E}+\frac{N}{G}\right),\ N_1 = \frac{1}{2}G_1\left(\frac{L}{E}+\frac{N}{G}\right)$$

b) Show that the curves on $\bar{S}$ corresponding to the lines of curvature of S are also lines of curvature on $\bar{S}$. **10**

OR

c) Show that the parallel surfaces of a minimal surface are surfaces for which **10**

Ra + Rb = constant, where $Ra = \frac{1}{Ka}$ & $Rb = \frac{1}{Kb}$

d) Show that the sphere is only the surface all points of which are umbilics, by using Codazzi equations. **10**

5. a) Define Orthogonal trajectories. **5**

b) Show that the geodesics on a right circular cylinder are helices. **5**

c) Fid L, M, N for the sphere **5**

$r = (a\cos u\cos v,\ a\cos u\sin v, a\sin u)$

Where us is the latitude & v is the longitude.

d) State the fundamental existence theorem for surfaces. **5**

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